14th International Conference on Applications of Statistics and Probability in Civil Engineering, ICASP14
Dublin, Ireland, July 9-13, 2023

Network Analysis of Systems of Systems Models for Complex Infrastructure Systems

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ABSTRACT: Complex infrastructure systems, such as transportation and communication networks, consist of many interconnected and interdependent elements that exhibit behavior that is difficult to predict. Researchers have developed System of Systems (SoS) approaches to model and understand the behavior of such complex systems. However, creating SoS models requires combining multiple complex models to describe the system's overall behavior, leading to many open questions about the influence of specific parameters on the results, how uncertainties propagate through the system, and what level of model detail is required. To address these challenges, this paper proposes a network approach to study the properties of computational SoS models for infrastructure systems. The proposed methodology utilizes network analysis to categorize the system and identify critical components, providing insights into the complexity and uncertainty of the system. Furthermore, theoretical concepts from network analysis are used to demonstrate how uncertainties propagate through the system and identify critical model parameters. The approach is applied to a multi-hazard transportation problem for a road network located in Switzerland. The results show that the network representation of SoS models provides a powerful tool for analyzing the behavior of infrastructure systems and can provide insights into how uncertainties propagate and which system components are critical. This approach can help researchers derive further knowledge about complex systems, provide a means for making informed decisions regarding system design and development, and help gain new insights into the behavior of SoS models.

1. INTRODUCTION

Complex systems comprise many interconnected and interdependent elements that exhibit behavior that is difficult to predict. Infrastructure systems like transportation and communication networks are prime examples of complex systems. They consist of many different parts that are embedded in space and interact with their environment, making them highly susceptible to natural hazards and other external factors (Hackl and Adey, 2019b). Infrastructure systems can experience complex dynamics, and as a result, the failure of even a single component in these systems can have a cascading impact on the entire system, even at remote locations (Hackl and Adey, 2019a).

To model and understand the behavior of complex infrastructure systems, researchers have developed "Systems of Systems" (SoS) approaches that focus on modeling the interactions and dependencies among the individual components and subsystems (Hall et al., 2016; Thacker et al., 2017; Zorn et al., 2020). This modeling framework captures the behavior and interactions of multiple interdependent infrastructure systems or subsystems and the relationships between them. It involves a hierarchical and modular approach that considers the system as a collection of interconnected systems. Each subsystem is modeled separately and then integrated into a more extensive (network) model that accounts for the dependencies and interdependencies between them. SoS models aim to provide a holistic view of the infrastructure system and its dynamic behavior under various operating conditions, including failures, natural hazards, and other disruptions (Hackl et al., 2018). They are typically used for assessing the resilience of infrastructure systems, identifying vulnerabilities and risks, and developing strategies to improve system performance and robustness (Sanderson et al., 2022).

However, creating SoS models requires combining multiple complex models to describe the system's overall behavior. This leads to many open questions, such as how influential specific parameters are for the results, how uncertainties propagate through the SoS, what the most important parameters are, and what level of model detail is required. Additionally, running large-scale SoS models is computationally intense, making probabilistic and sensitivity analyses very costly. Additionally, model extensions are expensive and require careful consideration. Therefore, new insights into these complex system modeling approaches are necessary to derive further knowledge.

To address these challenges, this paper proposes a network approach to study the properties of computational SoS models for infrastructure systems. Using simple network measures, the system can be categorized and critical components identified, providing insights into the complexity and uncertainty of the system. Furthermore, theoretical concepts from network analysis are used to demonstrate how uncertainties propagate through the system and identify the critical model parameters. This approach can help to derive further knowledge about complex systems, provide a means for making informed decisions regarding system design and development, as well as help researchers to gain new insights into the behavior of SoS models.

2. METHODOLOGY

In this work, network analysis is utilized to describe the mathematical relationships among the System of Systems (SoS) models for infrastructure systems. Networks are a powerful tool for modeling complex systems as they reduce the system's complexity into a tractable mathematical representation where properties of interest are represented as nodes and their relationships among each other as edges. Nodes and edges can have additional attributes such as labels, weights, or other types which can represent features of the real-world system. Furthermore, networks are not limited to a particular system but can be applied very generally.

In this work, networks are used to describe the mathematical relationships among the underlying physical models of the SoS representation. Nodes represent parameters, values, and equations, while edges represent how they are connected to each other. For example, a simple equation ($Eq\ X$) such as $Eq\ X:\ A+B=C$ can be represented in the network as nodes A, B, C, and $Eq\ X$ connected by edges, where the edge from A and B to $Eq\ X$ to C represents the mathematical relationship.

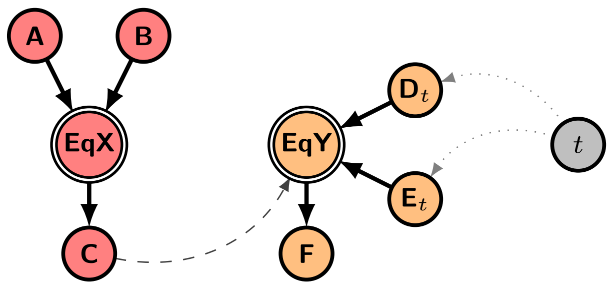


Figure 1: Example network representation. The red model, $A+B=C$, is connected with the orange model, $Eq\ y$. variables D and E depend on the variable t.

A multi-network approach is used to distinguish between different hierarchies within the SoS models. The edges in the network can represent different types of relationships, including model internal relationships based on physical equations, edges that are used as input to other system models, and edges that represent common variables that appear over multiple models. The nodes within a model are labeled together to identify the model they belong to.

The outcome of this methodology is a network representation of the SoS's physical models in terms of nodes and edges. One limitation of this approach is that only functional relationships are considered, and no distinction is made about the mathematical operators. Nonetheless, the network representation of SoS models provides a powerful tool for analyzing the behavior of infrastructure systems and can provide insights into how uncertainties propagate and which system components are critical.

3. APPLICATION

The methodology presented in this work is applied to a System of Systems (SoS) model proposed by Hackl et al. (2018). In the paper, the authors studied a multi-hazard transportation problem for a road network located in Switzerland to quantify the risk and resilience against flooding and landslides. The model considers both temporal and spatial changes throughout the hazard event, as well as the time needed for restoration.

The SoS model proposed by Hackl et al. (2018) is based on simple physical modules with a limited set of input parameters given in their paper's appendix. The SoS model is comprised of nine interdependent model categories, including rainfall, runoff, flood, landslide, object fragility, object functionality, traffic, direct costs, and indirect costs, with the aim to capture the complex dynamics of transportation infrastructure during a hazard event and the associated costs of interruptions of service.

The outlined methodology is used to derive a network representation of this SoS model. Since the simulation is sequential, a directed network representation is used, i.e., the edges are directed to capture the flow of information from one variable/module to the next. The nodes in the network represent the different parameters, values, and equations used in the SoS model, while the edges represent the relationships among them. A tenth category is introduced to address general variables that are used throughout the modeling process, such as DTM grid cells and time.

Applying this network-based approach, a network with 140 nodes and 265 edges could be derived from the SoS's physical models illustrated in Figure 2.

The resulting network model provides the basis for gaining insight into the SoS model's complexity and allows for identifying critical components and relationships. Simple network measures such as in or out-degree or betweenness centrality can be used to categorize the system and determine important properties. Uncertainty propagation can also be theoretically studied using the network diffusion model, and critical components of the system can be identified, which will be explored in the following sections.

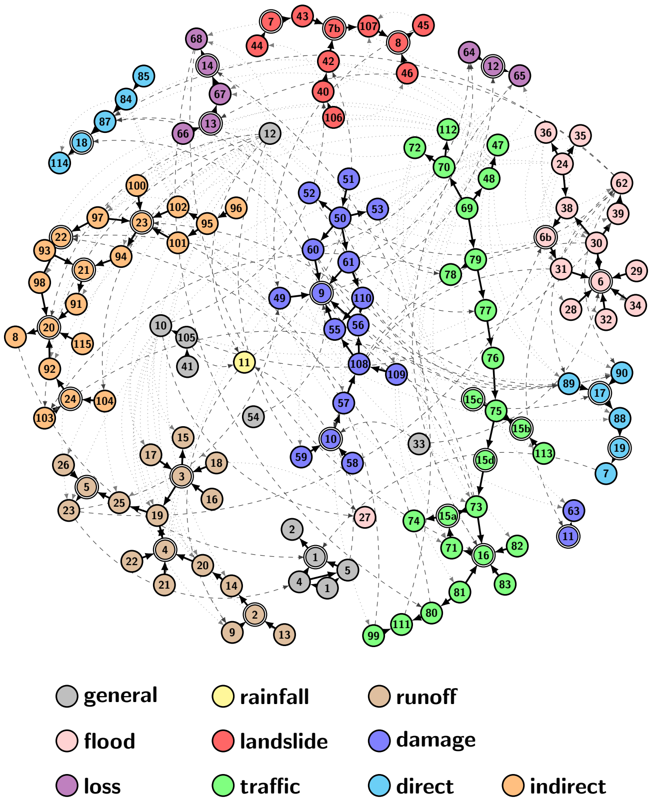


Figure 2: Network representation of the SoS model.

3.1. *Community detection*

Community detection in network science is the process of identifying groups of nodes within a network that are more densely connected to each other than to the rest of the network. These groups of nodes are called communities or clusters. Community detection is a fundamental task in network analysis, as it can reveal a network's underlying structure and organization and provide insights into the function and behavior of the system it represents. Identifying communities in an arbitrary network can pose computational challenges as the number of communities, if existent, is usually uncertain, and their sizes and densities may vary. For this analysis, modularity maximizationNewman (2004) was used. It involves partitioning a network into communities or clusters that have a higher density of connections within themselves compared to connections between different communities.

Figure 3 displays the identified communities in the network. A total of seven communities were identified, with a good fit observed between the community assignment and the different physical models, given that the community assignment is solely based on the topology. Some models were split into multiple communities, particularly the input and output nodes of the models. Notably, the traffic model was partitioned into two parts, with the network and origin-destination component of the traffic model (yellow) forming one community and the link flow (light green) forming another community. The link flow was also found to be in the same community as the costs for travel prolongation. Similar behavior was observed for other models.

The application of community detection is of significant interest as it provides insights into which models are closely linked in terms of interactions and shared variables, thereby guiding the consideration of which models should be analyzed together rather than independently.

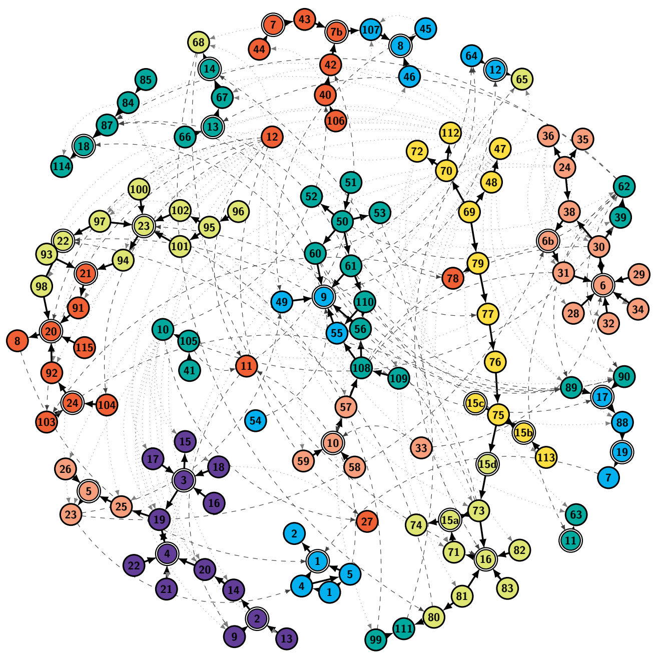


Figure 3: Detected communities.

3.2. *Node Centrality*

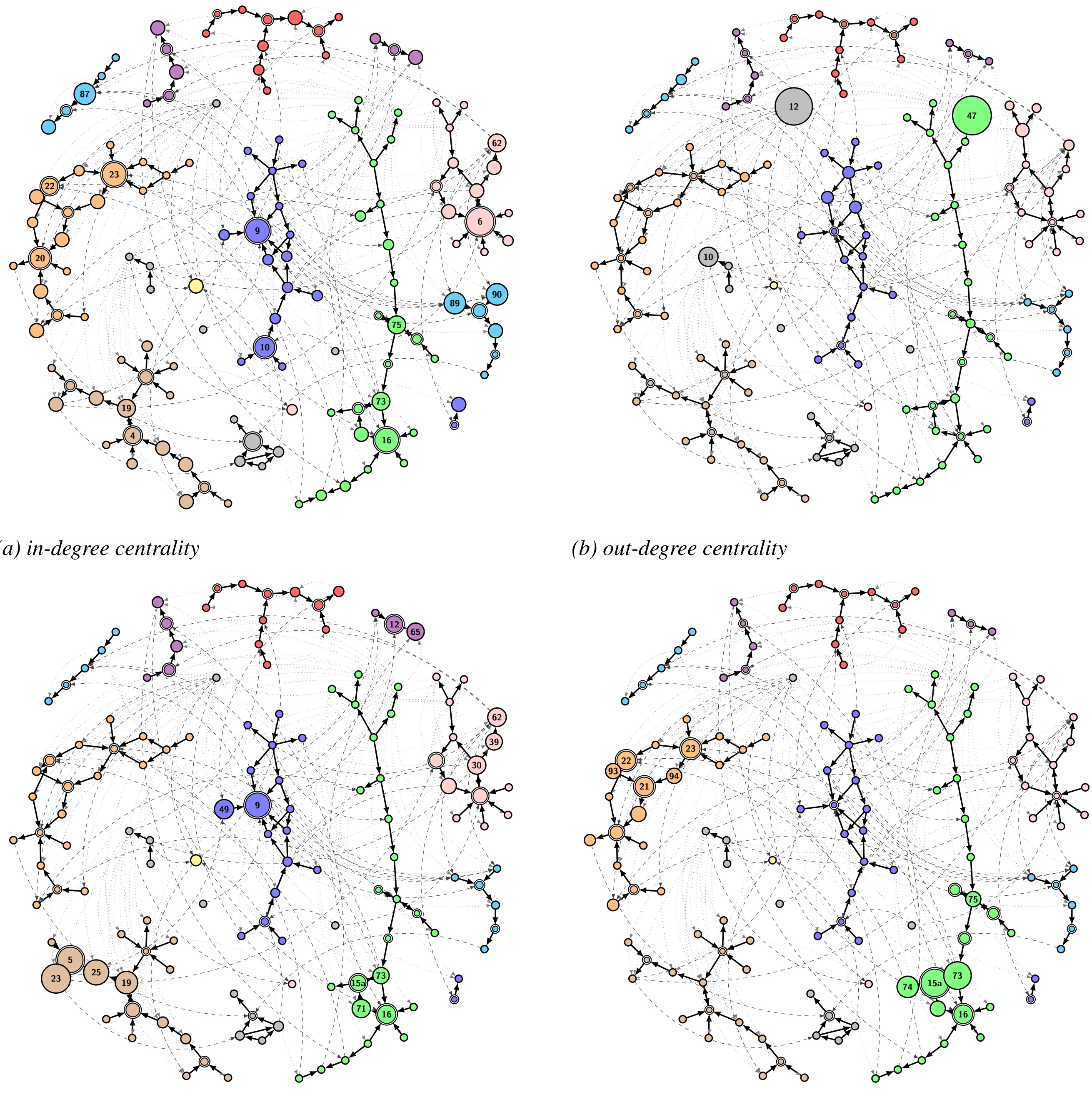
Node centrality is a concept in network science that measures the relative importance of a node within a network. It reflects the idea that some nodes in a network are more central or more important than others. Centrality is a crucial concept in understanding the structure and function of networks, as it can reveal which nodes are likely to have the most influence or play the most critical roles in the network. There are several types of node importance measures, each capturing a different aspect of node centrality. In this work, the following centralities are considered:

In-degree centrality is defined as the number of incoming edges or connections to a node in a directed network. Nodes with high in-degree centrality are considered more important or influential in the network, as they receive more connections or information from other nodes. In-degree centrality is often used to identify key nodes or hubs in networks, as well as to understand patterns of information flow within the network. It is particularly useful in applications such as social network analysis, where nodes may represent individuals and edges may represent social ties or relationships.

Out-degree centrality is a measure of the number of edges that are directed outwards from a node in a directed network. It is the number of outgoing links or edges from a node. In other words, it measures the extent to which a node is connected to other nodes in the network by sending information or resources to them. Nodes with high out-degree centrality are typically considered important in terms of dissemination or influence in a network.

Betweenness centrality quantifies how often a node lies on the shortest path between other pairs of nodes in the network. Nodes with high betweenness centrality are crucial for the overall connectivity and efficiency of the network, as they are responsible for maintaining communication and facilitating the transfer of information or resources between different groups of nodes. Hence, betweenness centrality can help identify important nodes that act as bottlenecks or bridges in a network.

Eigenvector centrality is a measure that captures both the number and quality of a node's connections, where the quality is measured by the centrality of the nodes to which it is connected. A node with high eigenvector centrality is connected to other nodes that are themselves highly central and may be important for spreading influence or controlling the flow of information in the network.



(a) in-degree centrality *(b) out-degree centrality*

(c) betweenness centrality *(d) eigenvalue centrality*

Figure 4: Node impotency measures.

They may be more likely to have a more significant impact on the overall structure or behavior of the network.

Figure 4 illustrates the four centrality measures applied to the network, with the top five most important nodes listed in Table 1. The hydraulic model (Node Eq6) and the object fragility model (Node Eq9) have the highest number of input pa-

Table 1: Top five most important nodes

Rank	in-deg	out-deg	betw	eigenv
1	Eq6	47	23	Eq15a
2	Eq23	12	Eq5	73
3	Eq16	10	Eq9	74
4	Eq9	24	25	Eq23
5	90	61	19	Eq21

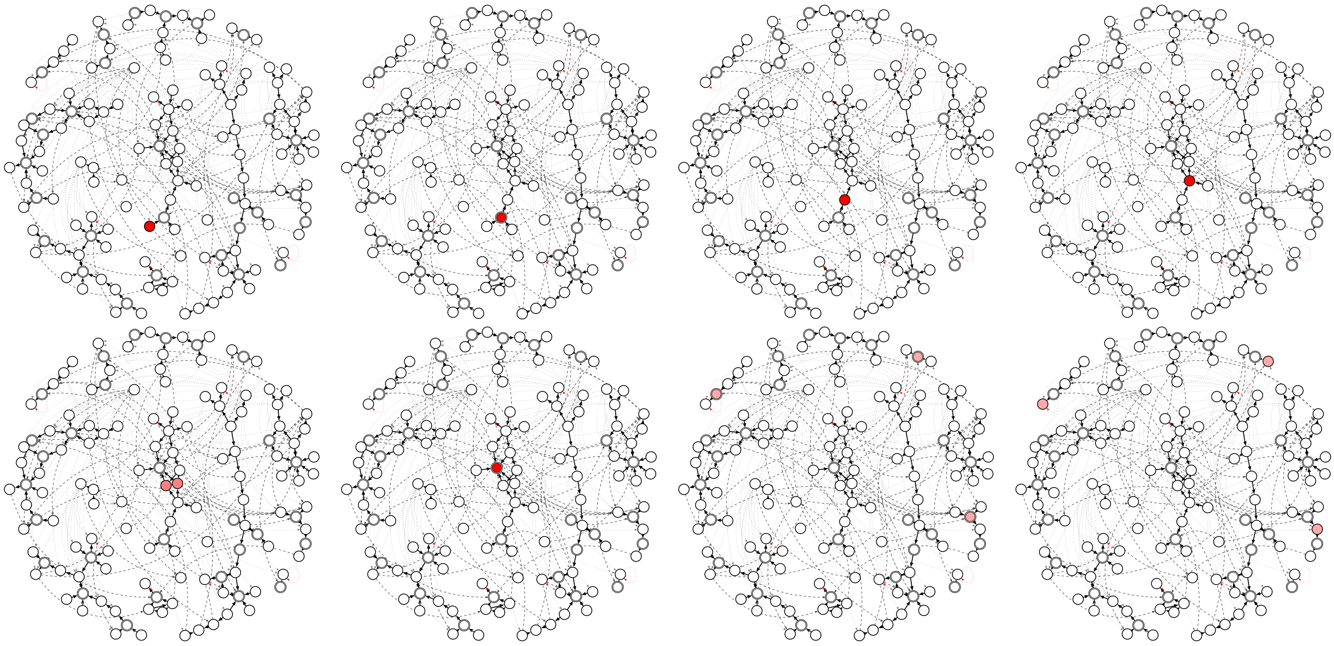


Figure 5: Diffucion process over time for node 59 (bridge pier width).

rameters. In contrast, the time variable (Node 12) and infrastructure objects (Node 47) are the most connected nodes, with 24 and 23 relationships, respectively. Betweenness centrality analysis revealed that the hydrograph (Node 23) and the corresponding runoff model (Node Eq5) have the highest centrality, followed by the object fragility model (Node Eq9). Eigenvalue centrality analysis identified clusters of highly central nodes around the user equilibrium assignment model (Eq15a) and the indirect costs for traveling models (Eq21, Eq22, Eq23).

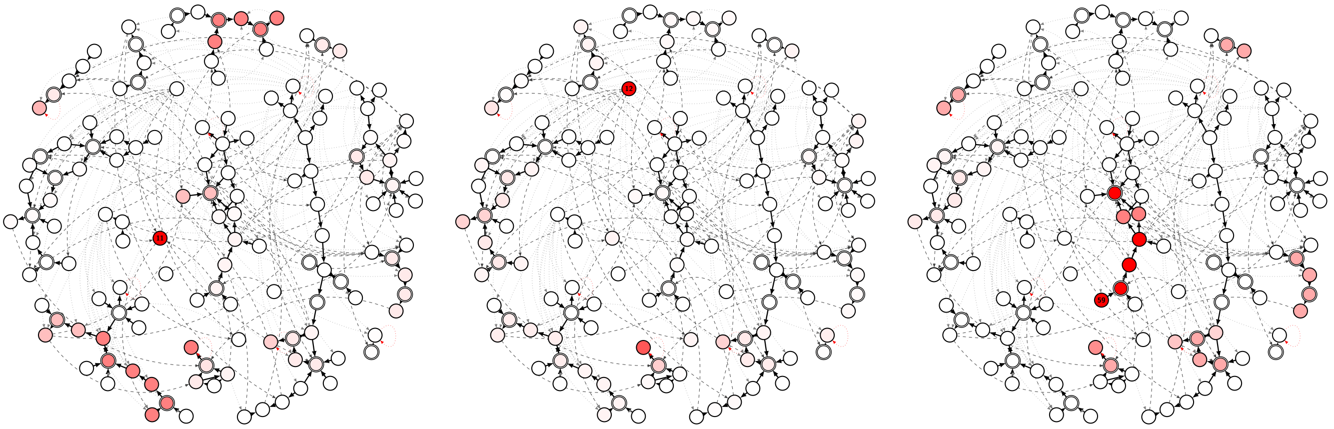
3.3. Uncertainty propagation via diffusion

Diffusion on networks for uncertainty quantification is a method that leverages the structure of a network to propagate uncertainty from input parameters to model outputs. The method involves simulating the diffusion of information or influences through the network, where each node in the network represents a model variable or parameter, and the edges represent their dependencies or relationships.

The diffusion process starts with the input nodes, which are assumed to have a probability distribution that captures the uncertainty or variability in their values. The probability distribution is then propagated through the network using a diffusion model, such as a random walk or a diffusion equation, which considers the network topology and the strength of the relationships between nodes. The diffusion process converges to a stationary distribution, representing the output nodes' probability distribution, and provides a measure of uncertainty or variability in the model predictions.

The diffusion on networks method has several advantages for uncertainty quantification compared to traditional methods, such as Monte Carlo simulation or sensitivity analysis. First, it can handle high-dimensional models with many input variables, as the complexity of the problem is reduced by exploiting the network structure. Second, it can capture the effect of correlations and dependencies between variables, which are often neglected in traditional methods. Third, it can provide a measure of the importance of each variable or parameter in the model based on its contribution to the output uncertainty. This information can be used for model calibration, optimization, or design.

The diffusion process depicted in Figure 5 demonstrates information propagation over time for a chosen node. At the initial time step $t = 0$, the information is located at its source node and shown in red. As the diffusion process progresses, the in-



(a) Node 11 (precipitation) *(b) Node 12 (time)* *(c) Node 59 (bridge pier width)*

Figure 6: Aggregated visiting probabilities.

formation is distributed equally among the successive nodes. The intensity of the colors in Figure 5 represents the node's influence by the initial node, with darker colors indicating a higher level of influence. As the diffusion process continues, a stationary state will be reached.

Figure 6 depicts the aggregated temporal network for the diffusion processes associated with Node 11 (precipitation), Node 12 (time), and Node 59 (bridge pier width). The precipitation variable is utilized as the source event to initiate flooding and landslides, and thus it exhibits the most significant influence on the runoff and landslide model. The time variable is input to several other nodes, as discussed in Section 3.2, and therefore diffuses evenly across all models. On the other hand, the bridge pier width variable has a strong local impact on the damage model, whereas other models are less affected.

4. CONCLUSIONS

In conclusion, the study of complex infrastructure systems requires a holistic view of the system's behavior and interactions under various operating conditions, including failures, natural hazards, and other disruptions. The Systems of Systems (SoS) approach provides such a framework by modeling the interactions and dependencies among individual components and subsystems. However, creating SoS models requires combining multiple complex models, leading to many open questions about the importance of specific parameters, how uncertainties propagate through the SoS, what the critical parameters are, and what level of model detail is required.

To address these challenges, this paper proposes a network approach to study the properties of computational SoS models for infrastructure systems. The proposed methodology uses network analysis to reduce the system's complexity into a tractable mathematical representation, where properties of interest are represented as nodes and their relationships among each other as edges. This approach allows the system to be categorized and critical components identified, providing insights into the complexity and uncertainty of the system. Furthermore, the proposed approach can help researchers to gain new insights into the behavior of SoS models and to make informed decisions regarding system design and development.

The methodology presented in this paper is applied to a System of Systems (SoS) model for a multi-hazard transportation problem in Switzerland. The SoS model is based on simple physical modules with a limited set of input parameters, and the proposed network approach provides a network representation of the SoS model. The network representation allows the identification of critical components, provides insights into how uncertainties propagate and can help assess the system's risk and resilience against flooding and landslides.

Despite the promising results presented in this paper, there are some limitations that should be considered. Firstly, the methodology proposed in this work is based on simplifications and abstractions of the underlying physical models of infrastructure systems. This means that some aspects of the system behavior may be lost or not fully captured by the network approach, particularly when dealing with highly nonlinear or complex models. Secondly, the network approach relies on the assumption that the relationships among the different components of the system can be represented as a network, which may not always be the case. Additionally, the proposed methodology has only been applied to a single case study, and its effectiveness in other contexts or for other types of infrastructure systems remains to be explored. Finally, while the network approach can help to identify critical components and parameters of the system, it may not provide specific information on how to improve the system's performance or reduce its vulnerability. Therefore, further research is needed to address these limitations and fully realize the potential of the proposed methodology.

One area of future research is to extend the methodology to include non-functional relationships, such as logical operations, which could provide more detailed information about the model's behavior. Additionally, the network approach can be used to optimize system performance and identify critical components by exploring different configurations of the system's parameters. Another possible direction for future research is to extend the methodology to account for uncertainty and variability in the physical models and parameters, as well as incorporate feedback mechanisms that capture the system's dynamic behavior over time. Additionally, the methodology can be used to compare different SoS models and assess their effectiveness in capturing the behavior of infrastructure systems. Furthermore, this approach can be applied to other types of complex infrastructure systems, such as communication networks or power grids, to analyze their behavior and identify critical components. Overall, the proposed work provides a promising foundation for further research in the field of infrastructure systems modeling and analysis.

In conclusion, the proposed network approach can be a valuable tool for analyzing the behavior of complex infrastructure systems, identifying critical components, and assessing the system's risk and resilience under various operating conditions. The insights gained from this approach can inform decision-making processes and help improve infrastructure system design and development.

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